Nonlinear model order reduction for parametrized variational inequalities: application to crowd motion

Giulia Sambataro[1], Virginie Ehrlacher[†], Aline Lefebvre-Lepot [⋆] and Loïc Gouarin [∗]

[1] Inria Nancy Grand Est, Université de Strasbourg, France
[†] CERMICS, École Nationale des Ponts et Chaussées; Inria, MATHERIALS, Paris, France

[⋆] Université Paris Saclay, Université Paris Cité, ENS Paris Saclay, France

[∗] CMAP, CNRS, École Polytechnique, France

In this talk I will present a fully data-driven nonlinear model reduction approach to predict the solutions of time-dependent parametrized variational inequalities. In our present context, we make a specific focus on discrete contact problems. A prototypical example is the agent-based model proposed in [1] to describe crowd motion in the presence of obstacles. In this model, in a discrete time setting, the set of velocities of each agent in the crowd is the solution at each time step to a constrained least-squares optimization statement. The parametric variations of the problem (associated with the geometric configuration of the domain where the agents evolve) have a very strong impact on the variability of the solution, both in terms of positions of the agents and of contact forces between them, the latter being given by the Lagrange multipliers associated to non-interpenetration constraints. This variability leads to a slowly decaying Kolmogorov n-width of the solution manifold, motivating the use of nonlinear reduced-order models. Inspired by [2, 3], we develop a graph convolutional neural network-based architecture that learns a nonlinear reduced representation and its dynamics under geometric changes. Using convolutional latent dynamics with a sliding time window, the model enables autoregressive predictions. It achieves significant computational savings while maintaining accuracy comparable to linear methods, and is validated against projection-based approaches and more complex scenarios.

References

[1] B. Maury and J. Venel, *A discrete contact model for crowd motion.* In: ESAIM: Mathematical Modelling and Numerical Analysis. pp 145–168, 2011

[2] F. Pichi, B. Moya, and J.S. Hesthaven, *A graph convolutional autoencoder approach to model order reduction for parametrized PDEs.* In: Journal of Computational Physics 501, 2024.

[3] F. Regazzoni et al. *Learning the intrinsic dynamics of spatio-temporal processes through Latent Dynamics Networks.* In: Nat Commun 15, 2024.