Master internship

le cnam

Structure-preserving reduced order models for conservation laws

Context

Numerical simulation of **partial differential equations** plays a crucial role in understanding and predicting the behaviour of complex physical phenomena. High-fidelity (HF) numerical methods have been developed over the years, such as finite element method, finite volume method, and spectral methods. These methods, although very accurate, can be prohibitively expensive to simulate parametrised PDEs in multi-query contexts. In the last few decades, extensive research studies have been carried out on **reduced order models** (ROMs) [HPR22]. The main aim of ROMs is to reduce the computational cost while maintaining an acceptable level of accuracy, by reducing the dimensionality of the problem compared to HF methods.

In this project, we focus on the specific case of **conservation laws** [Tor13], which model advection-dominated problems and physical phenomena characterised by no dissipation. Classical ROMs are known to fail in this scenario. The main objective of this project is to advance the field of ROMs of conservation laws by conveying the structure-preserving capabilities of HF methods to ROMs, while retaining computational efficiency.

This internship will be part of the ANR-funded project SPARCL (Structure-Preserving Approach for Reduced order models of Conservation Laws). **Funding to pursue a PhD after the internship is available.**

Internship description

The aim of the internship is to develop ROMs that **preserve the positivity** of variables such as density or water height and the presence of sharp fronts and **discontinuities**.

These two characteristics are closely related: in fact, classical ROMs such as the ones based on proper orthogonal decomposition exhibit spurious oscillations in the neighbourhood of discontinuities. This can lead to both negative values and to smoothed out fronts.

The intern will initially work on scalar linear and nonlinear conservation laws: this will allow them to familiarise themselves with ROMs in a simple and less computationally demanding case. The objective of this first step is to define an efficient numerical strategy, that will then be applied to one-dimensional systems of conservation laws in the second part of the internship.

Required skills

We are looking for a last-year master's or engineering degree student in applied mathematics, engineering, scientific computing, computer science or equivalent.

The ideal candidate has a good knowledge of **numerical methods for PDEs** and of one or more **programming languages**.

Location and duration

This position is for a **5-month internship**, with a flexible **starting date in spring 2025**.

The internship will take place at the Conservatoire National des Arts et Métiers (CNAM), situated in Paris (2, rue Conté, 75003).

The intern will be part of the mathematical and numerical modelling (M2N) laboratory. Visits to the mathematical institute of Bordeaux are possible.

The internship reward is about 600€/month (plus partial refund of transports card).

How to apply

Please, send your application (CV, grades and a short motivation letter) to camilla.fiorini@lecnam.net and alessia.del-grosso@inria.fr.

Contacts

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References

[HPR22] Hesthaven, J. S., Pagliantini, C., & Rozza, G. (2022). Reduced basis methods for time-dependent problems. *Acta Numerica*, 31, 265-345.

[Tor13] Toro, E. F. (2013). Riemann solvers and numerical methods for fluid dynamics: a practical introduction. Springer Science & Business Media.